CSCI 210: Computer Architecture
Lecture 14: Digital Logic

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Slides from Cynthia Taylor

CS History: The Manchester Transistor Computer

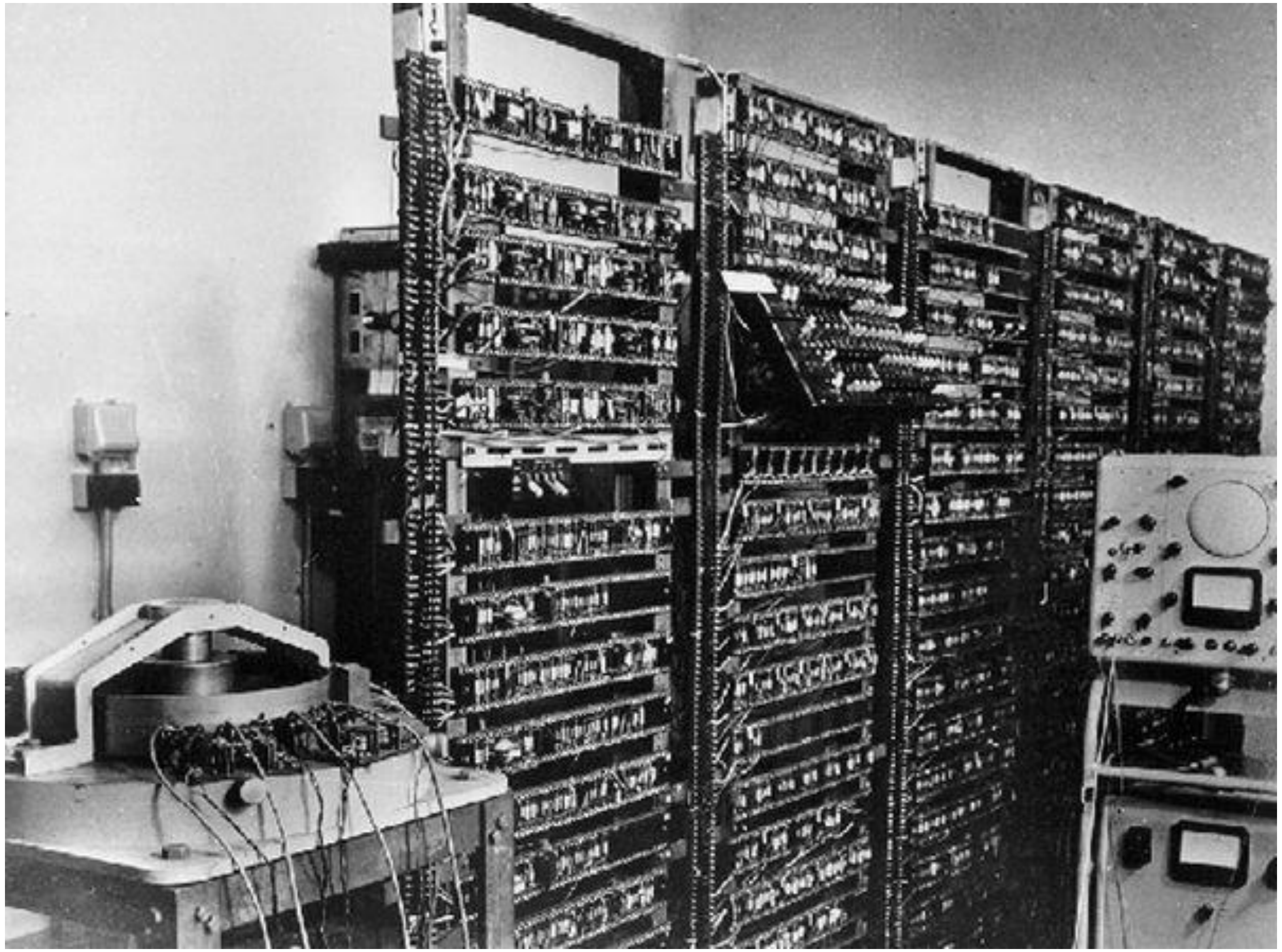


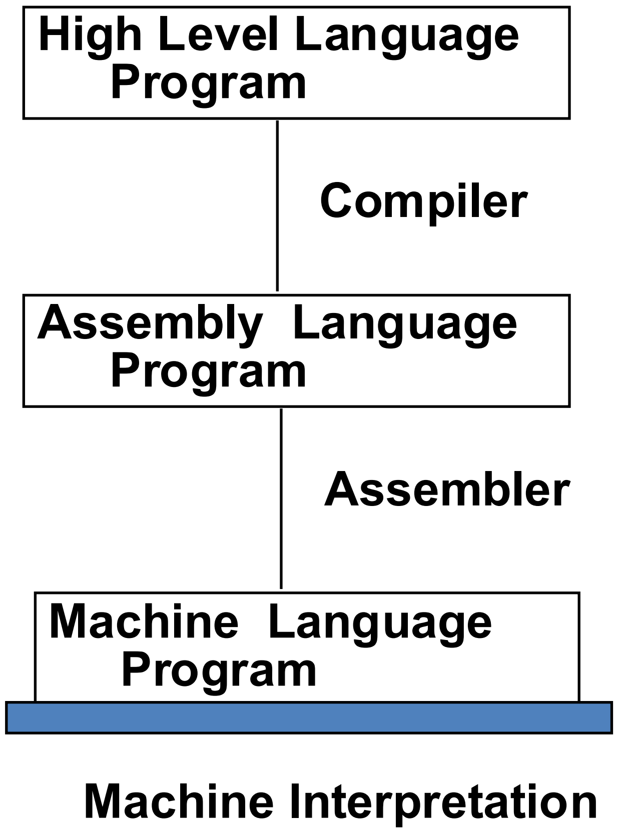
Image credit: The University of Manchester

- First computer to use transistors
- Developed at University of Manchester in 1953
- Problems with the reliability of early batches of transistors meant that its mean time between failures was about 90 minutes
- Still used vacuum tubes for its clock and memory, so not fully transistorized

Creating the Universe from 1 and 0

- We have seen how to build programs from assembly
- Now we’ll learn how we implement assembly language instructions using circuits

Machine Interpretation



```
temp = v[k];
v[k] = v[k+1];
v[k+1] = temp;
```

```
lw  $15,    0($2)
lw  $16,    4($2)
sw  $16,    0($2)
sw  $15,    4($2)
```

```
1000110001100010000000000000000
1000110011110010000000000000100
1010110011110010000000000000000
1010110001100010000000000000100
```

Machine does something!

A digital circuit is comprised of signals, gates, and wires

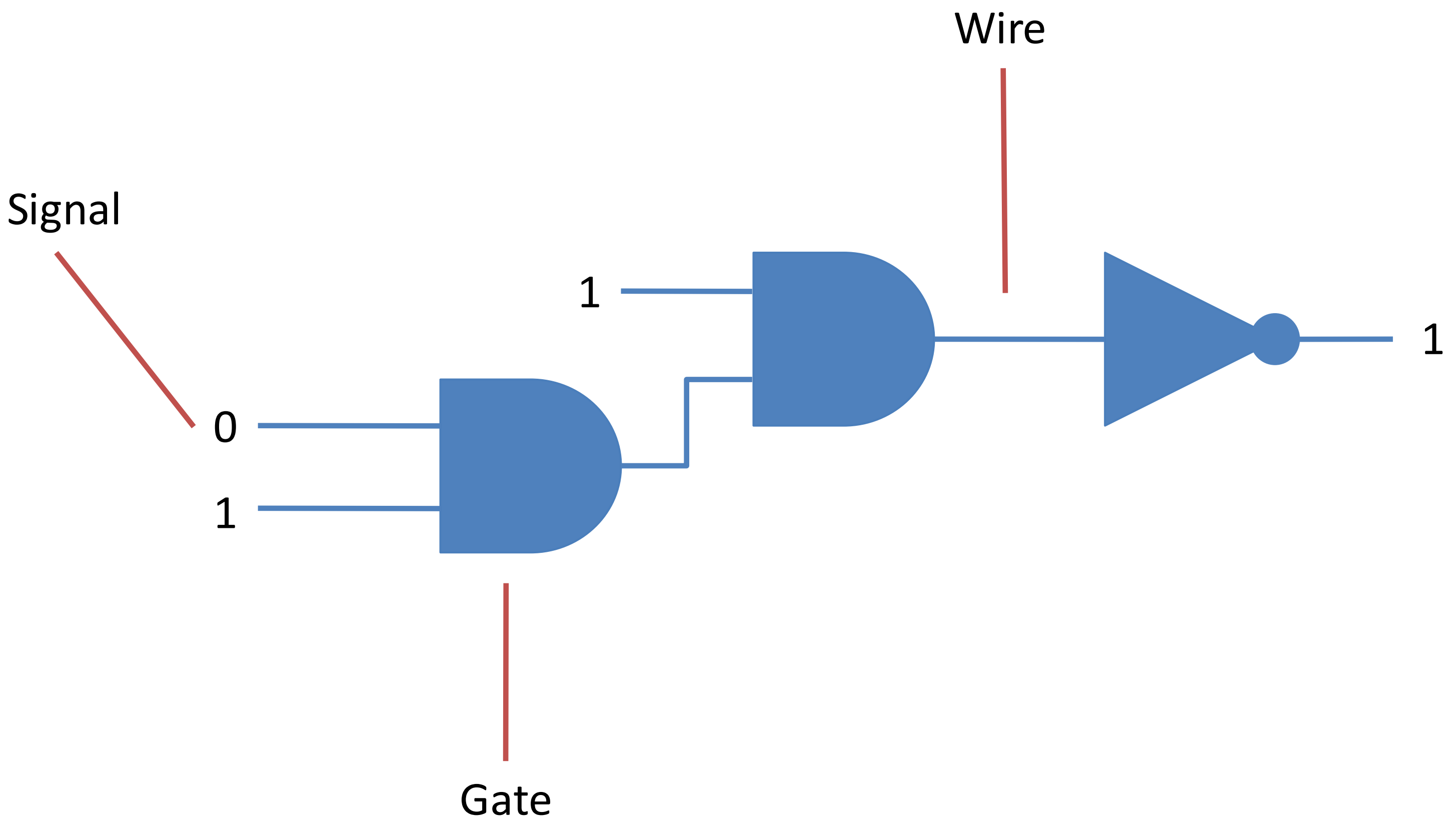
- Signals
  - Voltages applied to wires which generate electric current

- Binary signals are represented by different voltages:
  - 0: 0-1 volts
  - 1: 2-5 volts

A digital circuit is comprised of signals, gates, and wires

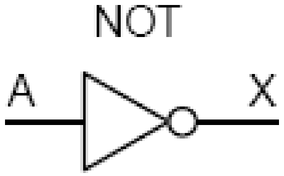
- Gates
  - Devices which perform operations on signals corresponding to basic logic operations: and, or, not, nand, nor, xor
  - Made out of transistors

- Wires
  - Lines over which signals are transmitted between gates

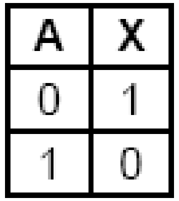

Wire
Signal
1
0
1
1
Gate

Representation of Logic Gates

- Symbol



- Truth Table

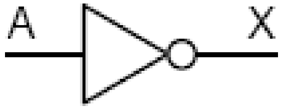


A	X
0	1
1	0

- Algebraic Representation: $\overline{A}$

- Connection to mathematical logic: 0 = False, 1 = True

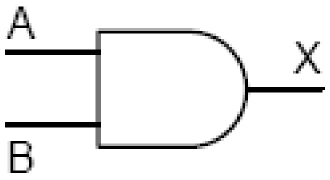
Not



A	X
0	1
1	0

- Inverts the input
- Algebraic representation: $\overline{A}$

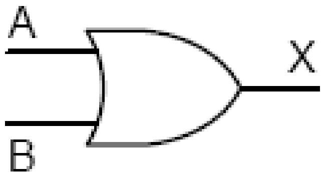
And



A	**B**	**X**
0	0	0
0	1	0
1	0	0
1	1	1

- Algebraic representation: AB or $A \cdot B$
- Notice that multiplication makes sense here: From the truth table, $X = \ A \cdot B$ with the usual meaning

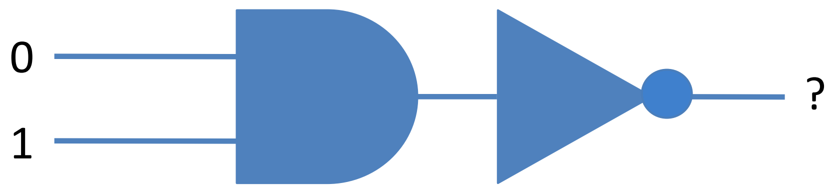
Or



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

- Algebraic representation: *A*+*B*
- Addition is max as in $A + B \; = \; \max(A, B)$
- We’ll see later that + and * obey some familiar properties of arithmetic (and some unfamiliar ones!)

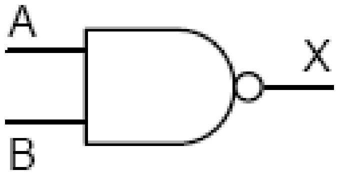
And and Not



A. 0

B. 1

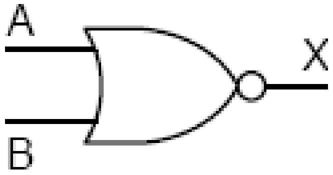
Nand



A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

- Algebraic representation: $\overline{AB}$
- Be careful with notation: $\overline{A \cdot B} = \overline{AB} \neq \overline{A} \cdot \overline{B}$

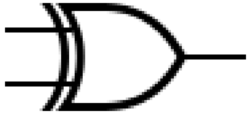
Nor



A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

- Algebraic representation: $\overline{A + B}$

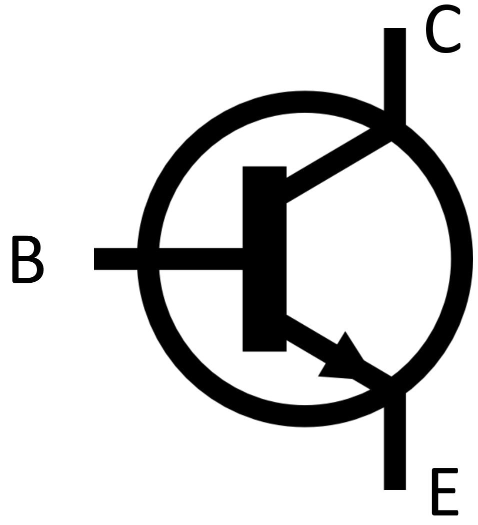
Xor

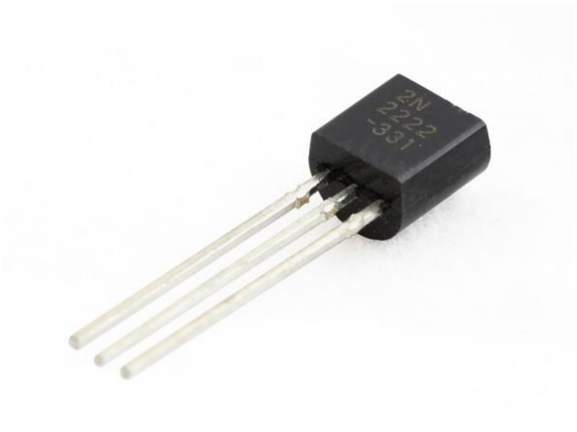


A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

- Algebraic representation: $A \oplus B$
- Programming languages tend to use A ^ B
- Fun fact: XOR is addition without a carry

Our Friend the Transistor

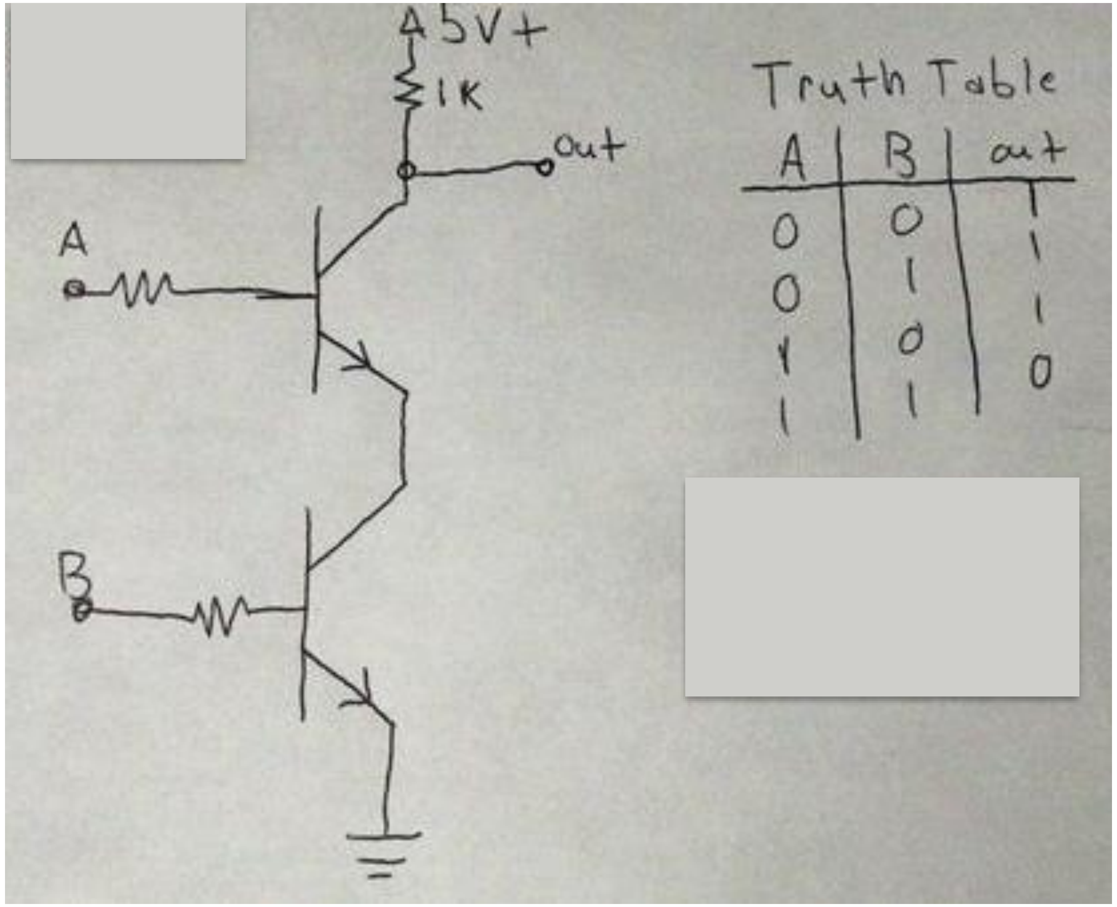




- The basic electronic component from which all gates are created; there are many types, this is an NPN bipolar junction transistor (NPN BJT)
- Applying a voltage to the base (B) (relative to the emitter (E)) allows current to flow from the collector (C) to the emitter
- This creates an on/off switch

Building gates out of switches

- Two inputs labeled A and B
- One output labeled out
- When A or B is 1, the other two electrodes (collector and emitter) are connected
- When A and B are both 1, out is connected to ground (logic value 0)
- When either A or B is 0, out is not connected to ground and current can flow from 5V to out



Truth Table

A	B	out
0	0	1
0	1	1
1	0	1
1	1	0

What Gate Does This Match?

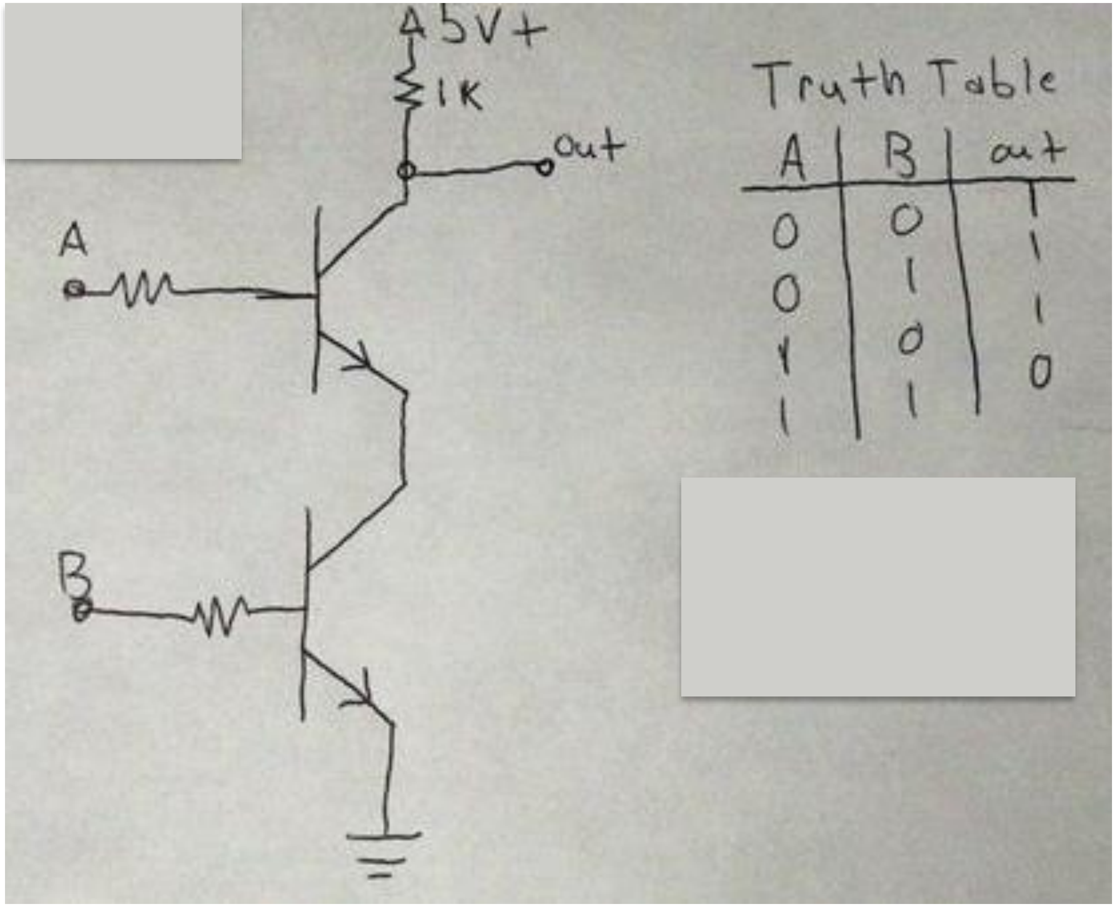
- If both A and B are high voltage (logical 1), out will be low voltage (logical 0)
- Otherwise, out is high voltage

A. AND

B. OR

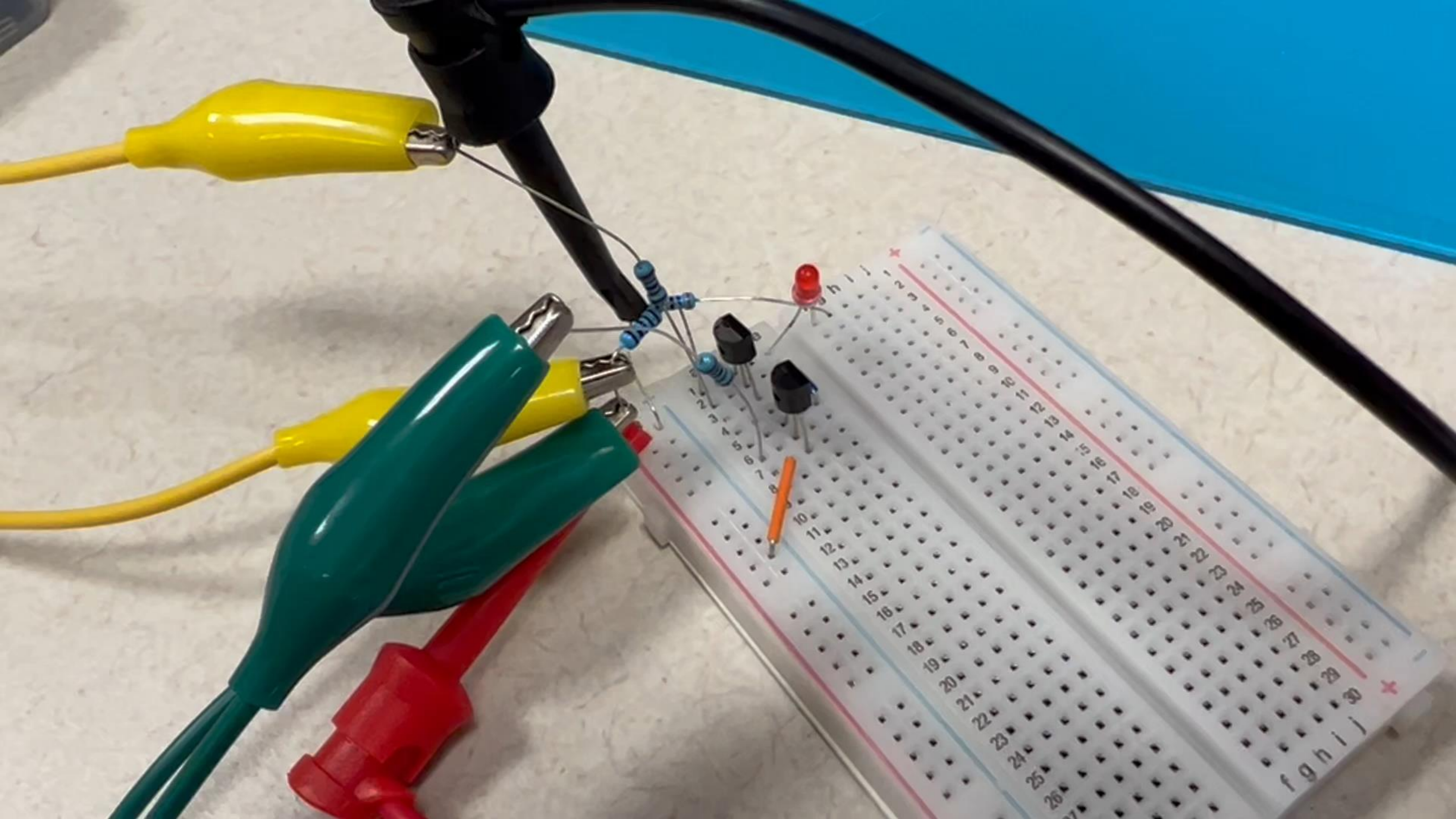
C. NAND

D. NOR



Truth Table

A	B	out
0	0	1
0	1	1
1	0	1
1	1	0


f g h i j
+

All Other Gates Can Be Created From NAND

Not

And

Which is equivalent to *A* OR *B*?

A. *A* NAND *B*

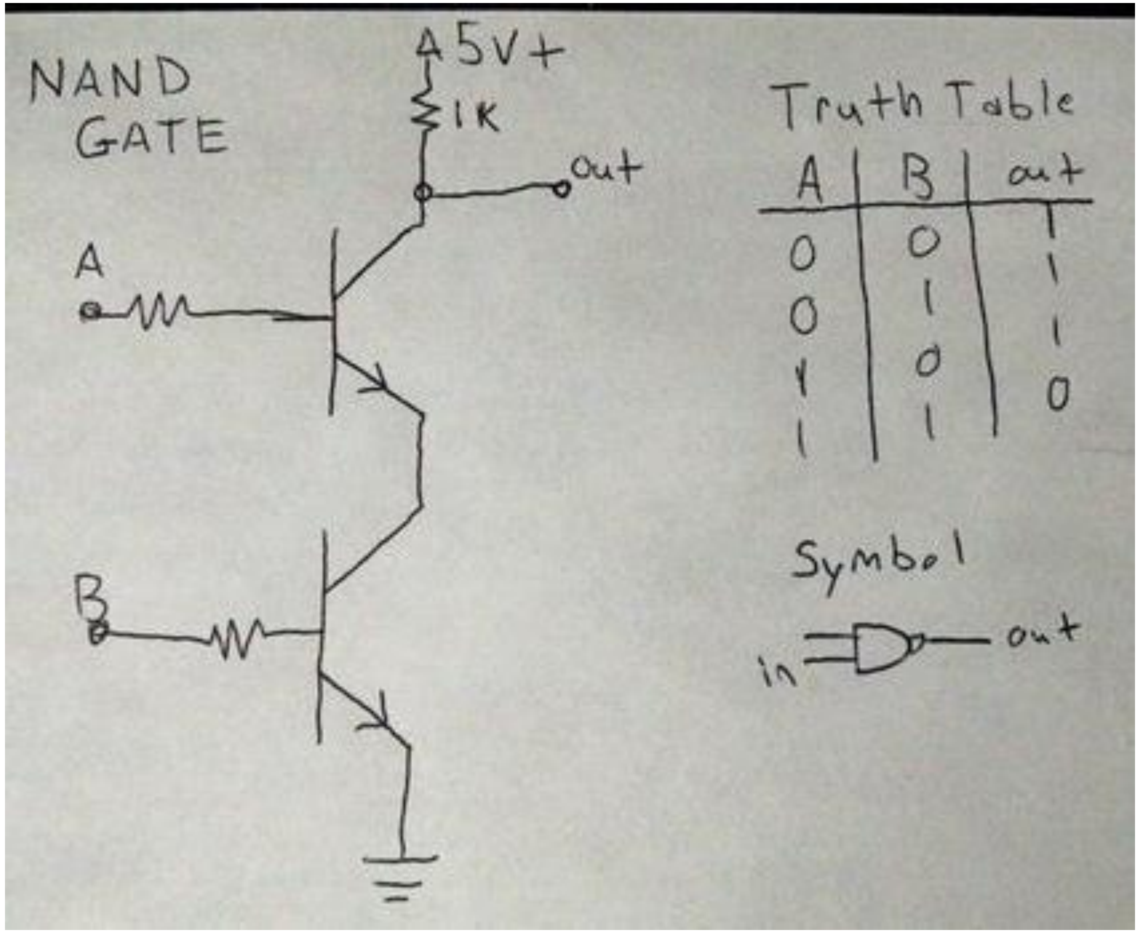
B. NOT (*A* NAND *B*)

C. (NOT *A*) NAND (NOT *B*)

D. NOT ((NOT *A*) NAND (NOT *B*))

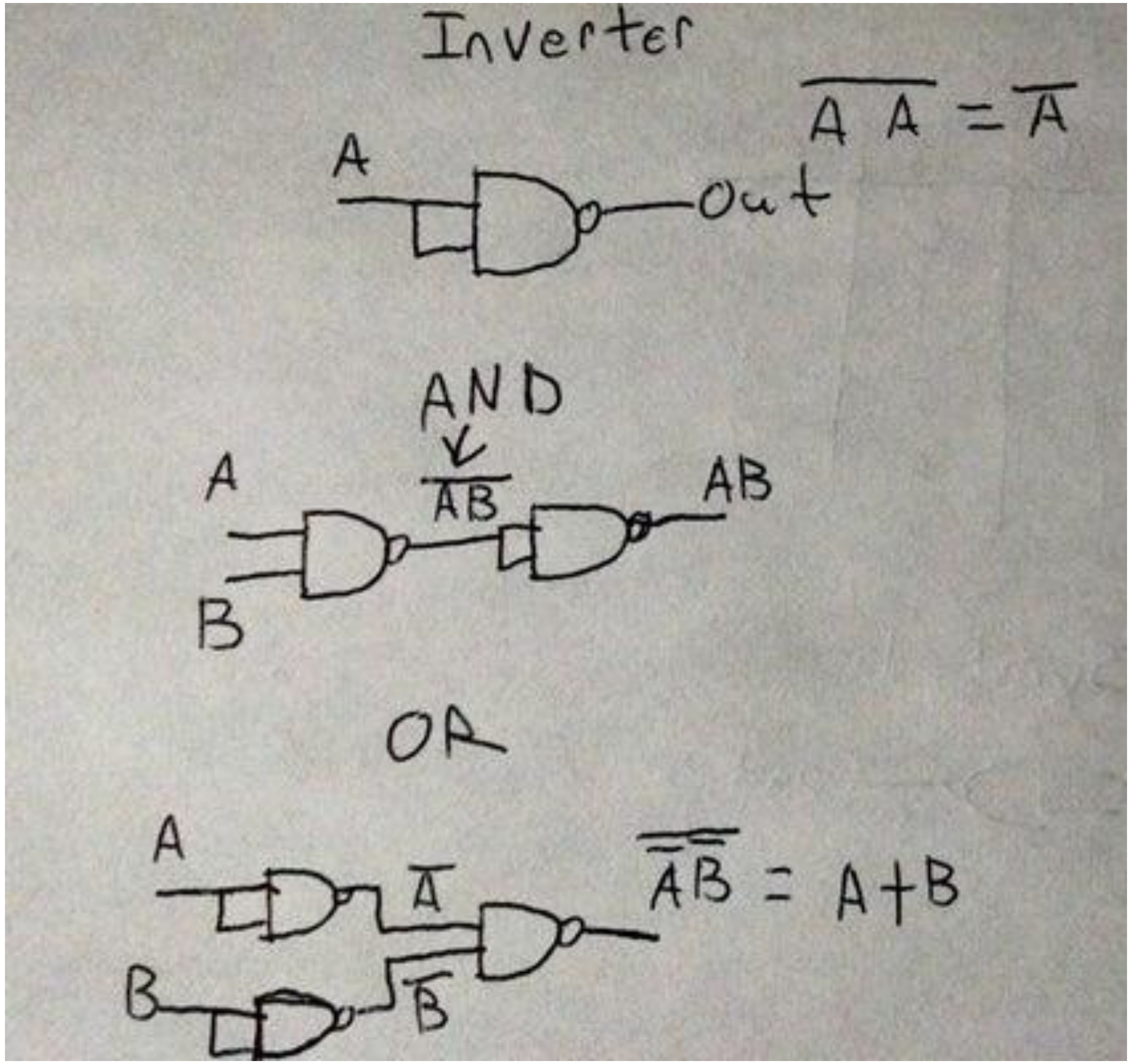
E. None of the above

Putting them together



Truth Table

A	B	out
0	0	1
0	1	1
1	0	1
1	1	0


Inverter
$\overline{A\ A} = \overline{A}$
A
Out
AND
$\overline{AB}$
A
B
AB
OR
A
$\overline{A}$
B
$\overline{B}$
$\overline{\overline{A}\,\overline{B}} = A+B$

Images from: https://www.instructables.com/Build-a-NAND-gate-from-transistors/

All Gates Can Also Be Created from NOR

- NOR and NAND are universal gates
  - All gates can be created from them

- You will show this in Problem Set 5

Which column completes the truth table for

$$F = \overline{X} \cdot (Y + Z)\,?$$

X	Y	Z	A	B	C	D
0	0	0	0	0	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	0	1	0	1
1	1	0	0	1	0	1
1	1	1	0	1	0	1

Groups: Draw circuit diagram for

$$F = \overline{X} \cdot (Y + Z)$$

$$F = \overline{A} + (B(AC + \overline{AB}))$$

Truth Table

A	B	C	AC	$\overline{AB}$	$AC + \overline{AB}$	$B(AC + \overline{AB})$	F
0	0	0					
0	0	1					
0	1	0					
0	1	1					
1	0	0					
1	0	1					
1	1	0					
1	1	1					